

This question paper contains 4 printed pages]

Roll No.

--	--	--	--	--	--	--	--	--	--

No. of Question Paper : 88

Unique Paper Code : 32351101

I

Name of the Paper : Calculus

Name of the Course : B.Sc. (H) Mathematics

Semester : I

Duration : 3 Hours

Maximum Marks : 75

Write your Roll No. on the top immediately on receipt of this question paper.)

All the sections are compulsory.

All questions carry equal marks.

Use of non-programmable scientific calculator is allowed.

### Section I

(Attempt any four questions from Section I)

If  $y = \log(x + \sqrt{x^2 + 1})$ , show that :

$$(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + n^2y_n = 0.$$

Sketch the graph of the function

$$f(x) = \frac{3x - 5}{x - 2}$$

by determining all critical points, intervals of increase and decrease, points of relative maxima and minima, concavity of the graph, inflection points and horizontal and vertical asymptotes.

P.T.O.

( 2 )

3. Evaluate :  $\lim_{x \rightarrow 0} (e^x - 1 - x)^x$ .
4. Given the cost  $C(x) = \frac{1}{8}x^2 + 5x + 98$  of producing  $x$  units of a particular commodity and the selling price  $p(x) = \frac{1}{2}(75 - x)$  when  $x$  units are produced. Determine the level of production that maximizes profit.
5. Sketch the graph of  $r = \sin 2\theta$  in polar coordinates.

### Section II

(Attempt any four questions from Section II)

6. Obtain the reduction formula for

$$\int \sec^n x \, dx.$$

Use it to evaluate  $\int \sec^6 x \, dx$ .

7. Find the volume of the solid generated by revolving the region enclosed by  $y = x$ ,  $y = 2 - x^2$  and  $x = 0$  is revolved about the  $x$ -axis.

8. Use cylindrical shells method to find the volume of the solid generated when the region enclosed by  $y = 2x - x^2$  and  $y = 0$  is revolved about  $y$ -axis.

9. Show that the arc length of the curve  $y = \cosh x$  between  $x = 0$  and  $x = \log 2$  is  $3/4$ .

10. Find the area of the surface generated by revolving the curve  $y = \sqrt{9 - x^2}$ ,  $-1 \leq x \leq 1$ , about  $x$ -axis.

## Section III

(Attempt any *three* questions from Section III)

11. Find the equation of parabola having axis  $y = 0$  and passing through the points  $(3, 2)$  and  $(2, -3)$ .
12. Find the equation of ellipse with foci  $(1, 2)$  and  $(1, 4)$  and minor axis of length 2.
13. Describe and sketch the graph of the conic

$$x^2 - 4y^2 + 2x + 8y - 7 = 0.$$

Label the vertices, foci and asymptotes to the graph.

14. Rotate the coordinate axes to remove the  $xy$ -term in the equation

$$31x^2 + 10\sqrt{3}xy + 21y^2 - 144 = 0.$$

Identify the resultant conic

## Section IV

(Attempt any *four* questions from Section IV)

15. Given the vector functions

$$\vec{F}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$$

and

$$\vec{G}(t) = \frac{1}{t}\mathbf{i} - e^t\mathbf{j}$$

verify that

$$\lim_{t \rightarrow 1} [\vec{F}(t) \times \vec{G}(t)] = [\lim_{t \rightarrow 1} \vec{F}(t)] \times [\lim_{t \rightarrow 1} \vec{G}(t)].$$

P.T.O.

## Section III

(Attempt any *three* questions from Section III)

11. Find the equation of parabola having axis  $y = 0$  and passing through the points  $(3, 2)$  and  $(2, -3)$ .
12. Find the equation of ellipse with foci  $(1, 2)$  and  $(1, 4)$  and minor axis of length 2.

13. Describe and sketch the graph of the conic

$$x^2 - 4y^2 + 2x + 8y - 7 = 0.$$

Label the vertices, foci and asymptotes to the graph.

14. Rotate the coordinate axes to remove the  $xy$ -term in the equation

$$31x^2 + 10\sqrt{3}xy + 21y^2 - 144 = 0.$$

Identify the resultant conic.

## Section IV

(Attempt any *four* questions from Section IV)

15. Given the vector functions

$$\vec{F}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$$

and

$$\vec{G}(t) = \frac{1}{t}\mathbf{i} - e^t\mathbf{j}$$

verify that

$$\lim_{t \rightarrow 1} [\vec{F}(t) \times \vec{G}(t)] = [\lim_{t \rightarrow 1} \vec{F}(t)] \times [\lim_{t \rightarrow 1} \vec{G}(t)].$$

P.T.O.

16. A velocity of particle moving in space is

$$\vec{V}(t) = t^2 \hat{i} - e^{2t} \hat{j} + \sqrt{t} \hat{k}.$$

Find the particle's position as a function of  $t$  if the position at time  $t = 0$  is  $\vec{R}(0) = \hat{i} + 4\hat{j} - \hat{k}$ .

17. A shell is fired at ground level with a muzzle speed of 280 ft/s and at an elevation of  $45^\circ$  from ground level :

- (i) Find the maximum height attained by the shell.
- (ii) Find the time of flight and the range of the shell.

18. Find the tangential and normal components of the acceleration of an object that moves with position vector

$$\vec{R}(t) = \cos t \hat{i} + \sin t \hat{j} + t \hat{k}.$$

19. Find the curvature  $\kappa(t)$  for the curve given by the vector equation

$$\vec{R}(t) = 4 \cos t \hat{i} + 4 \sin t \hat{j} + t \hat{k} \quad (0 \leq t \leq 2\pi).$$